Improving Reduction Techniques in Pseudo-Boolean Conflict Analysis (Extended Abstract)

Orestis Lomis
DTAI, KU Leuven, Belgium

Jo Devriendt
Nonfiction Software, Belgium

Hendrik Bierlee
DTAI, KU Leuven, Belgium

Tias Guns
DTAI, KU Leuven, Belgium

Abstract

Constraint satisfaction problems appear in many real world applications, such as nurse scheduling, AI planning, circuit verification, among others [1]. Over the past decennia many solving technologies have been proposed and studied both for modelling and solving such problems. One of the candidate technologies is the pseudo-Boolean solving paradigm. It has been shown that the cutting planes proof system, used in pseudo-boolean (PB) solvers, is exponentially stronger than the resolution proof system on which SAT-solvers are built [3]. PB solvers leverage the cutting planes proof system to perform SAT-style conflict analysis during search. This process learns implied PB constraints, which can prune later parts of the search tree and is crucial to a PB solver's performance. A key step in PB conflict analysis is the *reduction* of a reason constraint, where this reason constraint caused a variable propagation that contributed to the conflict. While necessary, reduction generally makes the reason constraint less strong. Consequently, different approaches to reduction have been proposed, broadly categorised as division- or saturation-based, with the aim of preserving the strength of the reason constraint as much as possible.

We presented novel techniques to generate stronger reduced constraints in both division-based [2] and saturation-based [4] reduction methods. As in established work [5], we can indeed prove dominance relationships between the various reduction methods, which guarantee that reduced constraints obtained from one are at least as strong as those from another. The experiments show that stronger reduced constraints can improve the solver performance for different solvers and benchmarks, but not uniformly across all problems. While there are improvements on crafted knapsack benchmarks [6], and the competition decision benchmarks for *Exact*, we observe little difference on competition optimisation benchmarks [7].

Hence our theoretical results provide a better understanding of reduction methods and the freedom there is in reducing constraints before addition. Empirically there is a lesser understood relationship between the strength of the reduced reason constraint, the strength of the learned constraint after all iterations of constraint addition in the conflict analysis, and the effect of the learned constraints on solver performance. However, these relationships are complex, because they involve the reduction and resolution of multiple reason constraints with the conflict constraint. With the insights of the paper we also see avenues to strengthen the reduced constraints further. For example, in division-based reduction we can relax the requirements for reduction methods (as in saturation-based reduction), which could lead to a smaller divisor or an increase in the amount of so called superfluous and anti-superfluous literals that we detect using our techniques.

We also saw how some combinations of reduction techniques can be effective, but they use heuristics, e.g. to choose between division- and saturation-based reduction for specific constraints. These heuristics are much less studied and can have a big impact on empirical performance which deserves further study.

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References

1 Sam Buss and Jakob Nordström. Proof complexity and sat solving. In A. Biere, M. Heule, H. van Maaren, and T. Walsh, editors, *Handbook of Satisfiability*, pages 233–350. IOS Press, 2009.

2 Jan Elffers and Jakob Nordström. Divide and Conquer: Towards Faster Pseudo-Boolean Solving. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence*, pages 1291–1299, Stockholm, Sweden, July 2018. International Joint Conferences on Artificial Intelligence Organization. doi:10.24963/ijcai.2018/180.

3 Armin Haken. The intractability of resolution. *Theoretical Computer Science*, 39:297–308, 1985. Third Conference on Foundations of Software Technology and Theoretical Computer Science. URL: https://www.sciencedirect.com/science/article/pii/0304397585901446, doi:10.1016/0304-3975(85)90144-6.

4 Daniel Le Berre, Pierre Marquis, and Romain Wallon. On Weakening Strategies for PB Solvers. In Luca Pulina and Martina Seidl, editors, *Theory and Applications of Satisfiability Testing – SAT 2020*, pages 322–331, Cham, 2020. Springer International Publishing. doi:10.1007/978-3-030-51825-7_23.

5 Gioni Mexi, Timo Berthold, Ambros Gleixner, and Jakob Nordström. Improving conflict analysis in mip solvers by pseudo-boolean reasoning. *arXiv preprint arXiv:2307.14166*, 2023.

6 David Pisinger. Where are the hard knapsack problems? *Computers & Operations Research*, 32(9):2271–2284, 2005. URL: https://www.sciencedirect.com/science/article/pii/S030505480400036X, doi:10.1016/j.cor.2004.03.002.

7 Roussel, Olivier. Pseudo-Boolean Competition of 2024. https://www.cril.univ-artois.fr/PB24/.